

# Electric Dipole Moments of Neutron-Odd Nuclei

Takehisa Fujita<sup>1,\*</sup> and Sachiko Oshima<sup>1,†</sup>

<sup>1</sup>*Department of Physics, Faculty of Science and Technology, Nihon University, Tokyo, Japan*

The electric dipole moments (EDMs) of neutron-odd nuclei with even protons are systematically evaluated. We first derive the relation between the EDM and the magnetic moment operators by making use of the core polarization scheme. This relation enables us to calculate the EDM of neutron-odd nuclei without any free parameters. From this calculation, one may find the best atomic system suitable for future EDM experiments.

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*1. Introduction*– The discovery of the violation of time reversal invariance in fundamental interactions is one of the most important subjects in modern physics. For this purpose, the measurement of neutron EDM must be best suited for the confirmation due to its simplicity. However, the life time of neutron is rather short for accurate measurements, and therefore it is natural that the EDM measurement of nucleus should become the next target. Up to now, experimentally speaking, the nuclear EDM in neutral atomic systems must be the best for all, but theoretically it is a dangerous attempt since the nuclear EDM in neutral atomic systems must be mostly shielded by atomic electrons [1–3]. The main physical reason of the shielding should come from the basic interaction of the EDM since one starts from the following Hamiltonian density due to the Lorentz invariance as well as the gauge invariance

$$\mathcal{H}_{edm} = -\frac{i}{2}d^{(i)}\bar{\psi}^{(i)}\sigma_{\mu\nu}\gamma_5\psi^{(i)}F^{\mu\nu} \quad (1)$$

where  $F^{\mu\nu}$  denotes the electromagnetic field strength.  $d^{(i)}$  is the coupling constant of the EDM interaction for the corresponding fermion  $\psi^{(i)}$ , and here it corresponds to the nucleon field. This Hamiltonian density can be reduced to the non-relativistic EDM Hamiltonian

$$H_{edm} = -\mathbf{d}^{(i)} \cdot \mathbf{E} \quad (2)$$

where  $\mathbf{d}^{(i)}$  is written as

$$\mathbf{d}^{(i)} = d^{(i)}\boldsymbol{\sigma}^{(i)}. \quad (3)$$

If the fermion is neutron, then  $d^{(n)}$  is connected to the EDM of neutron  $d_n$  as  $d_n = d^{(n)}$ .

The main aim of the EDM study is to determine the finite value of  $d_n$  by experiments. Until now, the observed upper limits of the neutron EDM  $d_n$  are around [4, 5]

$$d_n \simeq (0.3 \pm 0.5) \times 10^{-25} \text{ e cm}, \quad \text{Smith et al.} \quad (4)$$

$$d_n \simeq (1.9 \pm 5.4) \times 10^{-26} \text{ e cm}, \quad \text{Harris et al.} \quad (5)$$

and they are still consistent with zero EDM of neutron. At the same time, the experimental efforts to measure the atomic EDM have also been performed [6–10]. In particular, the EDM of  $^{129}\text{Xe}$  have been measured and the observed upper limit of the Xe EDM value is found to be [6, 7]

$$d_{\text{Xe}} \simeq (0.7 \pm 3.3 \pm 0.1) \times 10^{-27} \text{ e} \cdot \text{cm}. \quad (6)$$

In addition, the upper limit of the EDM of Hg atomic system is obtained more accurately as [8, 9]

$$d_{\text{Hg}} \simeq (0.49 \pm 1.29 \pm 0.76) \times 10^{-29} \text{ e} \cdot \text{cm} \quad (7)$$

and one can see that the nuclear EDM measurements in these atomic systems are quite precise in comparison with the neutron EDM measurement.

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\*Electronic address: fffujita@phys.cst.nihon-u.ac.jp

†Electronic address: oshima@phys.cst.nihon-u.ac.jp

Last several years, the theoretical investigation of the nuclear EDM in neutral atomic systems has been made quite intensively [11–17]. By now, it is clarified that the extraction of the neutron EDM from the neutral atomic system is almost impossible due to the electron shielding mechanism which is quite a general physical process in all neutral systems. This is somewhat unfortunate since the EDMs of the neutral atomic system have been measured much more precisely than the ultra-cold neutron experiment by three orders of magnitude, but these EDMs may not be related to the neutron EDM because of the shielding mechanism.

However, a recent study indicates that the EDM of ionic systems can be directly related to the neutron EDM with some reduction factors [15–17]. In this case, the electron shielding mechanism becomes incomplete, and thus the EDM of ions can be described by the neutron EDM without having any further complications.

In this report, we discuss the EDM of the ionic systems in which two electrons are stripped off. In this case, the reduction factor becomes  $2/Z$ , and it should be noted that, if one makes ions where one electron is stripped off, then the reduction factor becomes  $1/Z$ , and so on. In this case, we assume that the atomic state should have the spin zero state which does not affect on the spin precession.

*2. EDM of Ions in shell model calculations*– In ions with two electrons stripped off, the nuclear EDM can be described as [16, 17]

$$d_A = \frac{2}{Z} \langle \Psi_0 | \sum_{i=1}^A d^{(i)} \sigma_z^{(i)} | \Psi_0 \rangle \quad (8)$$

where  $A$  and  $Z$  denote the mass number and proton number of nucleus, respectively.  $\Psi_0$  denotes the nuclear ground state. This is the nuclear EDM term which is escaped from the Schiff theorem.

The calculations of the nuclear EDM in ions can be reduced to the evaluation of the spin matrix elements. The shell model calculations of the spin operators have been made in many nuclear processes [18–20]. Here, the calculations can be carried out by making use of the experimental values of the nuclear magnetic moments [21]. This is clear since the magnetic moment operator in nucleus can be written as

$$\boldsymbol{\mu} = \mu_N \sum_{i=1}^A (g_s^{(i)} \mathbf{s}^{(i)} + g_\ell^{(i)} \boldsymbol{\ell}^{(i)}) = \mu_N \sum_{i=1}^A ((g_s^{(i)} - g_\ell^{(i)}) \mathbf{s}^{(i)} + g_\ell^{(i)} \mathbf{j}^{(i)}) \quad (9)$$

where  $\mu_N$  denotes the nuclear magneton, and hereafter, we set  $\mu_N = 1$ .  $g_s^{(i)}$  and  $g_\ell^{(i)}$  denote the g-factors of nucleon. For the neutron odd nucleus with even numbers of protons, we can evaluate the magnetic moment  $\mu_A$  in terms of the configuration mixing method [22] which is the modification only for the spin operators because there is no effect on the  $\mathbf{j}^{(i)}$  operators from the core polarization. Thus, the magnetic moment  $\mu_A$  can be written as [23]

$$\mu_A = \langle s_z \rangle (g_s^{(n)} + \delta g_s), \quad \langle s_z \rangle = \begin{cases} \frac{1}{2} & \text{for } j = \ell + \frac{1}{2} \\ -\frac{1}{2} \frac{2\ell-1}{2\ell+1} & \text{for } j = \ell - \frac{1}{2} \end{cases} \quad (10)$$

where  $\delta g_s$  denotes the effective  $g$ -factor which arises from the perturbation calculation and is given as

$$\delta g_s = -\frac{8}{3} \alpha_p N_p (g_s^{(p)} - 1) \frac{2\ell_p(\ell_p + 1)}{2\ell_p + 1} - \frac{8}{3} \alpha_n N_n g_s^{(n)} \frac{2\ell_n(\ell_n + 1)}{2\ell_n + 1}. \quad (11)$$

Here,  $\ell_p$  ( $\ell_n$ ) and  $N_p$  ( $N_n$ ) denote the angular momentum and the number of the core protons (neutrons) which can contribute to the configuration mixing.  $\alpha_p$  and  $\alpha_n$  are related to the residual nucleon-nucleon interaction, and for simplicity, we make them as free parameters so as to reproduce the observed magnetic moments of nucleus. From the property of the residual interaction, it is safe to assume the following relation between  $\alpha_p$  and  $\alpha_n$

$$\alpha_p = -3\alpha_n \equiv -3\alpha \quad (12)$$

where  $\alpha$  is the only free parameter. The experimental values of  $g$ -factors are given  $g_s^{(p)} = 5.585$  and  $g_s^{(n)} = -3.826$ . By making use of the value of  $\alpha$  determined from the observed nuclear magnetic moments, we can evaluate the EDM for the neutron odd nucleus with the mass number  $A$

$$d_A = \left( \frac{2}{Z} \right) \langle s_z \rangle \left( d_n - \frac{8}{3} \alpha_p N_p d_p \frac{2\ell_p(\ell_p + 1)}{2\ell_p + 1} - \frac{8}{3} \alpha_n N_n d_n \frac{2\ell_n(\ell_n + 1)}{2\ell_n + 1} \right) \quad (13)$$

where  $d_p$  denotes the proton EDM, and as one sees, this is the parameter free calculation. Since the observed magnetic moments can be reproduced by the configuration mixing method within 20 % errors, the evaluation of eq.(13) can be also reliable in the same level of accuracy.

In Table 1, we present the calculated results of the EDM for the neutron odd nucleus with even protons for wide ranges of nuclear chart. From this Table, one can hopefully find the best possible candidate for the EDM experiment with ions which should have the spin zero atomic state.

Table 1

Nucleus	$A$	$Z$	$J^P$	$\mu_A^{exp}$	$\alpha$	$d_A$
Be	9	4	$\frac{3}{2}^-$	-1.1778	0.0117	$0.23d_n + 0.063d_p$
C	13	6	$\frac{1}{2}^-$	0.7024	-0.0016	$-0.057d_n + 0.0037d_p$
O	17	8	$\frac{5}{2}^+$	-1.89379	0	$0.125d_n$
Mg	25	12	$\frac{5}{2}^+$	-0.8555	0.0047	$0.073d_n + 0.030d_p$
Si	29	14	$\frac{1}{2}^+$	-0.5553	0.0040	$0.060d_n + 0.033d_p$
S	33	16	$\frac{3}{2}^+$	0.6438	0.0025	$-0.034d_n - 0.011d_p$
Ar	39	18	$\frac{7}{2}^-$	-1.3	0.00046	$0.056d_n + 0.015d_p$
Ca	43	20	$\frac{7}{2}^-$	-1.3176	0.017	$0.034d_n$
Ni	61	28	$\frac{3}{2}^-$	-0.750	0.0018	$0.031d_n + 0.014d_p$
Zn	67	30	$\frac{5}{2}^-$	0.875	0.0090	$-0.021d_n - 0.0046d_p$
Ge	73	32	$\frac{9}{2}^+$	-0.879	0.0035	$0.031d_n + 0.014d_p$
Sr	87	38	$\frac{9}{2}^+$	-1.094	0.0038	$0.017d_n + 0.0014d_p$
Zr	91	40	$\frac{5}{2}^+$	-1.304	0	$0.025d_n$
Sn	119	50	$\frac{1}{2}^+$	-1.047	0.00012	$0.020d_n + 0.0076d_p$
Te	125	52	$\frac{1}{2}^+$	-0.889	0.0010	$0.016d_n + 0.0057d_p$
Xe	129	54	$\frac{1}{2}^+$	-0.778	0.0012	$0.015d_n + 0.0065d_p$
Ba	135	56	$\frac{3}{2}^+$	0.838	0.00052	$-0.0098d_n - 0.0016d_p$
Nd	143	60	$\frac{7}{2}^-$	-1.066	0.0023	$0.010d_n + 0.0007d_p$
Hg	201	80	$\frac{3}{2}^-$	-0.560	0.00092	$0.0097d_n + 0.0051d_p$

Table 1 shows the calculated values of the nuclear EDM  $d_A$  for odd-neutron nuclei in the  $A^{++}$  ionic states where two electrons are stripped off. The observed values of the nuclear magnetic moments and the calculated values of  $\alpha$  are also shown [24].

*3. Discussions and Remarks*– By now, we have learned that the nuclear EDM in neutral atomic systems must be mostly shielded by atomic electrons. Intuitively, electrons always block the penetration of the electric field inside nucleus, and unless one excites atomic electron states, one cannot find the penetration of the electric field inside nucleus. But, still, from the excitation of electrons, one can find only a very small flux of electric field inside nucleus.

This shielding mechanism can be well explained in the following fashion. In the Schiff screening, the nucleon EDM operator ( $\sum_i^A \mathbf{d}^i \cdot \mathbf{E}_{ext}$ ) cannot be seen because electrons trapped in the Coulomb field of  $A_0(r_j)$  always react as

$$\sum_{i=1}^A \sum_n \langle \psi_0 | \sum_{j=1}^Z \mathbf{d}^i \cdot \nabla A_0(r_j) | \psi_n \rangle \frac{1}{E_0 - E_n} \langle \psi_n | \sum_{k=1}^Z e \mathbf{r}_k \cdot \mathbf{E}_{ext} | \psi_0 \rangle + h.c. \quad (14)$$

which can be just rewritten as  $(-\sum_i^A \mathbf{d}^i \cdot \mathbf{E}_{ext})$ , and this cancels out completely the original nucleon EDM operator. Here,  $\psi_n$  denotes the atomic wave function. Now, one can easily see that this Schiff theorem cannot hold for the nuclear EDM in hydrogen-like atom with  $Z$  different from unity [17]. This is clear since the proof is based on the Coulomb field for electron, but the electric field on to the nuclear charge is produced by electron, and this is different from the  $A_0(r)$  potential. Therefore, unless the charge  $Z$  is equal to unity, the nuclear EDM survives while the electron EDM always vanishes to zero due to the Schiff theorem.

In addition, the cancellation due to the electron screening takes place for nuclear dipole operators ( $\sum_i^Z e \mathbf{R}_i \cdot \mathbf{E}_{ext}$ ) since the following higher order effects

$$\sum_{i=1}^Z \sum_n \langle \psi_0 | \sum_{j=1}^Z \frac{e^2 (\mathbf{r}_j \cdot \mathbf{R}_i)}{r_j^3} | \psi_n \rangle \frac{1}{E_0 - E_n} \langle \psi_n | \sum_{k=1}^Z e \mathbf{r}_k \cdot \mathbf{E}_{ext} | \Psi_0 \psi_0 \rangle + h.c. \quad (15)$$

can be reduced to  $(-\sum_i^Z e\mathbf{R}_i \cdot \mathbf{E}_{ext})$  which cancels out completely the original nuclear dipole operator [12–15]. Therefore, as long as we consider the nuclear EDM operator, we cannot observe the EDM of nucleon from the neutral atomic system. This proof is done at the level of operators for the nuclear variables, and therefore, it is impossible to extract the nuclear EDM from the neutral atomic systems. Here, we should note that the finite nuclear EDM in the neutral systems can be obtained only through the atomic excitation in the intermediate states where the nuclear state is taken to be in the ground state. In this case, one obtains the nuclear EDM [12, 15]

$$d_A = -\sum_n \frac{2e}{E_n - E_0} \langle \psi_0 | \sum_{i=1}^Z \left[ \mathbf{r}_i \cdot \sum_{j=1}^A \mathbf{d}^j \left( \frac{5}{2} - \frac{15}{2} \cos^2 \theta_{ji} \right) R_j^2 \right] \frac{e}{r_i^5} | \psi_n \rangle \langle \psi_0 | \sum_{i=1}^Z z_i | \psi_0 \rangle \simeq 2.4 \times 10^{-6} d_n. \quad (16)$$

This suppression factor of  $2.4 \times 10^{-6}$  is too small to carry out the EDM measurement so as to compete with the direct neutron measurement. In addition, the above estimation is made by using optimistic values for the parameters in the calculation. Also, one can easily see that the main reason of this suppression comes from the ratio between atomic radius  $a_0$  and nuclear radius  $R_0$ , and the factor is proportional to  $\sim (R_0/a_0)^2$  which is quite small. This suppression factor is indeed confirmed by the new calculation of Yoshinaga et al. [25] who carried out quite elaborate shell model calculations in Xe nucleus. Therefore, practically, there is no chance to observe the nuclear EDM in the neutral atomic system.

In order to overcome this difficulty, we should go on to the measurement of nuclear EDM in ionic systems as we show here. Therefore, one should strip a few electrons from atoms, and then one can measure the nuclear EDM which is directly related to the nucleon EDM. A question is, of course, as to how people can measure the nuclear EDM of ions which are certainly affected by the electric field  $\mathbf{E}_{ext}$ . This should be solved by experimentalists, and indeed Orlov et al. presented an interesting proposal to measure the EDM of ions in the storage ring experiments [26]. This type of experiments should become very important in future, but at this moment, we cannot claim anything further, and should wait for any EDM measurements from the storage ring experiments or any other EDM experiments of ionic systems.

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